SEVERAL TOPOLOGIES ON $P(\omega)$

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Winter School 2009.

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Forcing by $P(\kappa)$ does not add new sets. This makes them almost uninteresting (Am I right or not?)

$P(\kappa)$ with the product topology

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The space $\langle 2^\kappa, \tau_c \rangle$ is a zero-dimensional, homogenous, Hausdorff and compact space.

$\mathbb{B}$ with the sequential topology

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For a sequence x let

$$\limsup x = \bigwedge_{k\in\omega} \bigvee_{n\geq k} x_n \quad \liminf x = \bigvee_{k\in\omega} \bigwedge_{n\geq k} x_n$$

A sequence x algebraically converges to a point a ($\lambda_A(x) = a$) iff $\limsup x = \liminf x = a$

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The sequential topology τ_s is the maximal topology such that algebraic convergence implies topological convergence.

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$\langle \mathbb{B}, \tau_s \rangle$ is a sequential T_1 homogenous space

$\langle P(\kappa), \tau_s \rangle$

Topology No. 2

Theorem (B. Balcar, W. Glówczyński, T. Jech, 1998)

The space $\langle P(\kappa), \tau_s \rangle$ is:

- Hausdorff
- regular iff $\kappa = \omega$
- Fréchet iff $\kappa < \mathfrak{b}$
- sequentially compact iff $\kappa < \mathfrak{s}$
- compact iff $\kappa = \omega$
- zero-dimensional
- $\tau_c \subset \tau_s$
- $\tau_c = \tau_s$ iff $\kappa = \omega$

$\langle \mathbb{B}, \mathcal{O}^{\uparrow} \rangle$

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The maximal topology in which $\lambda^{\uparrow}$-convergence implies topological convergence is denoted by $\mathcal{O}^{\uparrow}$.

$\langle \mathbb{B}, \mathcal{O}^{\uparrow} \rangle$ is a sequential connected T_0 compact space, which is never T_1.

$\langle \mathbb{B}, \mathcal{O}^{\uparrow} \rangle$ and closed sets

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Question

Can we minimize a set $A \subset F$ such that $F = \bigcup_{b \in A}(b\uparrow)$?

$\langle \mathbb{B}, \mathcal{O}^{\uparrow} \rangle$, closed sets and *ccc*

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Theorem

If $\mathbb{B}$ is a *ccc* c.B.a., for each closed set F there holds

$$F = \bigcup_{b \in \mathrm{Min}(F)} (b\uparrow),$$

where $\mathrm{Min}(F)$ is the set of minimal elements of F.

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Example

If $\mathbb{B}$ is not a ccc c.B.a., then there exists strictly decreasing sequence $\langle a_\alpha : \alpha < \omega_1 \rangle$.

$$\overline{\{a_\alpha : \alpha < \omega_1\}} = \bigcup_{\alpha < \omega_1} (a_\alpha \uparrow)$$

but this set does not have minimal elements.

$\langle \mathbb{B}, \mathcal{O}^{\uparrow} \rangle$ and sets of form $\bigcup_{x \in X}(q_x \uparrow)$

Lemma

Let X be a non empty set and $\{q_x : x \in X\} \subset \mathbb{B}$.
Let $\tau = \{\langle \check{x}, q_x \rangle : x \in X\}$ and $F = \bigcup_{x \in X}(q_x \uparrow)$.
If $x \neq y \Rightarrow q_x \neq q_y$, then the following conditions are equivalent:

(a) q_x and q_y are incomparable for different $x, y \in X$;

(b) $\forall x, y \in X \ (x \neq y \Rightarrow ||\check{x} \in \tau \not\ni \check{y}|| > 0)$;

(c) $\{q_x : x \in X\} = \mathrm{Min}(F)$.

$\langle \mathbb{B}, \mathcal{O}^{\uparrow} \rangle$ and minimal elements

Example

A set of the form $\bigcup_{x \in X} (q_x \uparrow)$ must not be closed, even when $\{q_x : x \in X\}$ is the set of minimal elements.
If $\{q_x : x \in X\}$ is an infinite antichain then

$$\overline{\bigcup_{x \in X} (q_x \uparrow)} \supset \overline{\{q_x : x \in X\}} = \mathbb{B}$$

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Question

When is the set of a form $\bigcup_{x \in X}(q_x \uparrow)$, where $\{q_x : x \in X\}$ is the set of its minimal elements, closed in the space $\langle \mathbb{B}, \mathcal{O}^{\uparrow} \rangle$?

Subbase countably compact spaces

Subbase countably compact spaces

Definition (J. Gerlits, I. Juhász, Z. Szentmiklóssy, 2001.)

Let $\langle X, \mathcal{O} \rangle$ be a topological space.

If $\mathcal{S}$ is a subbase for the topology $\mathcal{O}$, the space $\langle X, \mathcal{O} \rangle$ is an $\mathcal{S}$-countably compact ($\mathcal{S}$-CC) space iff
$\forall A \in [X]^\omega \, \exists x \in X \, \forall S \in \mathcal{S} \, (x \in S \Rightarrow |S \cap A| = \omega)$.
x is $\mathcal{S}$-accumulation point

$\langle X, \mathcal{O} \rangle$ is a subbase countably compact (SCC) space iff there exists a subbase $\mathcal{S}$ for $\mathcal{O}$ such that $\langle X, \mathcal{O} \rangle$ is an $\mathcal{S}$-CC space.

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Each Lindelöf SCC space is compact.

Subbase countably compact spaces

Definition

Let $X \neq \emptyset$. $\mathcal{S} \subset P(X)$ is a T_1 *SCC subbase* iff

(i) $\bigcup \mathcal{S} = X$;

(ii) $\forall x, y \in X \ (x \neq y \Rightarrow \exists S \in \mathcal{S} \, (x \in S \not\ni y))$;

(iii) $\forall A \in [X]^\omega \, \exists x \in X \, \forall S \in \mathcal{S} \, (x \in S \Rightarrow |S \cap A| = \omega)$.

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Topology generated by a countable T_1 SCC subbase $\mathcal{S} = \{S_k : k \in \omega\}$ on X is T_1 Hausdorff compact topology on X

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If $\langle X, \mathcal{O}\rangle$ is a compact second-countable T_1 space, then $|X| \leq \omega$ or $|X| = \mathfrak{c}$.

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For $f : X \to P(Y)$ let $f^* : Y \to P(X)$ be defined by $f^*(y) = \{x \in X : y \in f(x)\}$.

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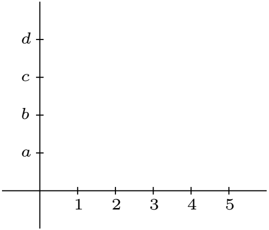

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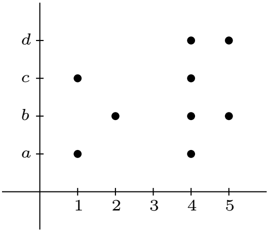

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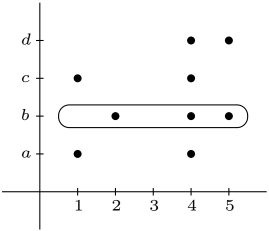

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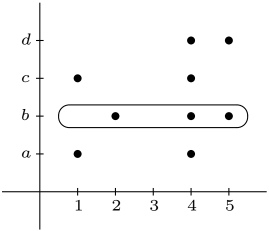

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If $f : \mathbb{N} \to \mathbb{N}$ such that $f(n) = \{k \cdot n : k \in \mathbb{N}\}$ then

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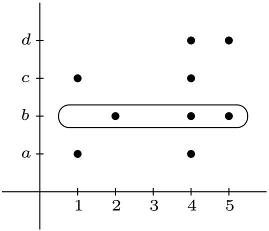

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If $f : \mathbb{N} \to \mathbb{N}$ such that $f(n) = \{k \cdot n : k \in \mathbb{N}\}$ then

$$f^* = \{k : k \text{ is a divisor of } n\}$$

$\langle \mathbb{P}(\omega), \mathcal{O}^{\uparrow} \rangle$

Topology No. 3

For a mapping $B : X \to P(\omega)$ by $\tau^B = \{\langle \check{x}, B_x \rangle : x \in X\}$ we denote the corresponding nice name for a subset of X.

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If G is a $P(\omega)$-generic filter over V and $S = B^*$, then $G = \{k\} \uparrow$ for some $k \in \omega$ and $(\tau^B)_G = S_k$.

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Lemma

Let $B : X \to P(\omega)$ be one-to-one mapping and $S = B^*$. If $F = \bigcup_{x \in X}(B_x \uparrow)$, then the following conditions are equivalent:

(a) Elements B_x, $x \in X$, are incomparable;

(b) $\forall x, y \in X \ (x \neq y \Rightarrow \exists k \in \omega \ (x \in S_k \not\ni y))$;

(c) $\{B_x : x \in X\} = \mathrm{Min}(F)$.

$\langle \mathbb{P}(\omega), \mathcal{O}^{\uparrow} \rangle$ and closed sets

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Lemma

If $B : X \to P(\omega)$, $S = B^*$, $\tau^B = \{\langle \check{x}, B_x \rangle : x \in X\}$ and $F = \bigcup_{x \in X}(B_x \uparrow)$ then the following conditions are equivalent:

(a) F is closed in $P(\omega)$;

(b) $\forall f : \omega \to X \ \exists x \in X \ B_x \subset \limsup \langle B_{f(n)} \rangle$;

(c) $\forall A \in [X]^\omega \ \exists x \in X \ 1 \Vdash \check{x} \in \tau^B \Rightarrow |\tau^B \cap \check{A}| = \check{\omega}$;

(d) $\forall A \in [X]^\omega \ \exists x \in X \ \forall k \in \omega \ (x \in S_k \Rightarrow |S_k \cap A| = \omega)$.

$\langle \mathbb{P}(\omega), \mathcal{O}^{\uparrow} \rangle$ and characterisation of closed stes

Theorem

If $B : X \to P(\omega)$, $S = B^*$ and $F = \bigcup_{x \in X} B_x \uparrow$ then the following conditions are equivalent:

(a) $F \in \mathcal{F}^{\uparrow} \setminus \{\omega\}$;

(b) $\mathcal{S} = \{S_k : k \in \omega\}$ is a T_1 SCC subbase (it generates some second countable compact topology on X);

(c) $\mathcal{S} = \{S_k : k \in \omega\}$ is a subbase for a T_1 compact second countable topology on X.

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Theorem

If $F = \bigcup_{B \in \mathrm{Min}(F)} B \uparrow$ is closed in $P(\omega)$ then $|\mathrm{Min}(F)| \leq \omega$ or $|\mathrm{Min}(F)| = \mathfrak{c}$.

Example

We will construct a closed set on $P(\omega)$ using the cofinite topology on ω. Let $[\omega]^{<\omega} = \{K_k : k \in \omega\}$.

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Then $B_n = \{k \in \omega : n \notin K_k\}$ and it generates the closed set

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Choosing another subbase, for instance, $\{S_k = \omega \setminus \{k\} : k \in \omega\}$, then $B_n = \omega \setminus \{n\}$ and we obtain the closed set

$$F = \bigcup_{n \in \omega} (\omega \setminus \{n\}) \uparrow = \{A \subset \omega : |\omega \setminus A| \leq 1\}.$$

Example

Let $\{A_n : n \in \omega\} \subset P(\omega) \setminus \{\emptyset\}$ be a family of disjoint sets, and let $B_n = \omega \setminus A_n$.

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Let $F = \bigcup_{n\in\omega} B_n \uparrow$.
B_n, $n \in \omega$, are incomparable and F is closed.

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Then $S_k = \{n \in \omega : k \in B_n\} = \{n \in \omega : k \notin A_n\}$.
If $k \in \omega \setminus \bigcup_{n\in\omega} A_n$, then $S_k = \omega$, otherwise, if $k \in A_n$, then $S_k = \omega \setminus \{n\}$.

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Let $F = \bigcup_{n \in \omega} B_n \uparrow$.
B_n, $n \in \omega$, are incomparable and F is closed.
Then $S_k = \{n \in \omega : k \in B_n\} = \{n \in \omega : k \notin A_n\}$.
If $k \in \omega \setminus \bigcup_{n \in \omega} A_n$, then $S_k = \omega$, otherwise, if $k \in A_n$, then $S_k = \omega \setminus \{n\}$.
We have obtained the same subbase as in the second part of previous example.

$\langle \mathbb{P}(\omega), \mathcal{O}^{\downarrow} \rangle$

Topology No. 4

Using the a priori operator $\lambda^{\downarrow}(x) = (\liminf x) \downarrow$ we obtain the topology denoted by $\mathcal{O}^{\downarrow}$, which is, in sense od Boolean algebras, dual topology to $\mathcal{O}^{\uparrow}$

$\langle \mathbb{P}(\omega), \mathcal{O}^* \rangle$

Topology No. 5

Definition

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Theorem (Not just in $P(\omega)$)

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Question

When does it hold $\mathcal{O}^* = \tau_s$?

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Theorem (Not just in $P(\omega)$)

$$\lim_{\mathcal{O}^*} = \lim_{\tau_s} .$$

Question

When does it hold $\mathcal{O}^* = \tau_s$?

Theorem

On Boolean algebra $P(\omega)$ there holds $\mathcal{O}^* = \tau_s = \tau_c$.

J. Gerlits, I. Juhász, Z. Szentmiklóssy,
Subbase countable compactness,
Studia Sci. Math. Hungarica 38 (2001) 225–231.

B. Balcar, W. Główczyński, T. Jech,
The sequential topology on complete Boolean algebras,
Fund. Math. 155 (1998) 59–78.